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DIOPHANTINE ANALYSIS.

62. Proposed by JOHN M. ARNOLD, Crompton, R. I.

Find, if possible, four square numbers in arithmetical progression.

I. Solution in Imaginaries by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

I have not, thus far, succeeded in obtaining a solution in real numbers, but the following in imaginaries.

Let $(x-y)^2$, x^2+y^2, $(x+y)^2$, $(x+y)^2+2xy$ be the numbers.

Let $x=m^2-n^2$, $y=2mn$.

$\therefore$ $(m^2-n^2-2mn)^2$, $(m^2+n^2)^2$, $(m^2-n^2+2mn)^2$, $(m^2-n^2+2mn)^2+4mn(m^2-n^2)$.

The last is a square when $m=n$; let $m=n+1$.

$\therefore$ $(1-2n^2)^2$, $(2n^2+2n+1)^2$, $(2n^2+4n+1)^2$, $4n^4+24n^3+32n^2+12n+1$.

Assume, $4n^4+24n^3+32n^2+12n+1=(2n^2+6n+k)^2$; also $4n^4+24n^3+(32+a^2)n^2+(12+2ab)n+1+b^2=(an+b)^2$.

$\therefore$ $a^2=4+4k$, $6+ab=6k$,. $k^2=1+b^2$.

$\therefore$ $k^3-8k^2+17k-10=0$. $\therefore$ $k=1$ or 2 or 5.

$\therefore$ $n=0$ or $-\frac{1}{2}\pm\frac{1}{2}\sqrt{(-11)}$ or -1.

$\therefore$ $25\pm12\sqrt{(-11)}$, 25, $25\mp12\sqrt{(-11)}$, $25\mp24\sqrt{(-11)}$, or $[6\pm\sqrt{(-11)}]^2$, $(5)^2$, $[6\mp\sqrt{(-11)}]^2$, $\{\sqrt{[(25+\sqrt{6961})/2]}\mp\sqrt{[(25-\sqrt{6961})/2]}\}^2$.

II. Comment by JOSIAH H. DRUMMOND, LL. D., Counselor at Law, Portland, Me.

Let $x^2-2xy+y^2$, x^2+y^2, $x^2+2xy+y^2$, $x^2+4xy+y^2$ be the four numbers ; two of them being squares, we have to make $x^2+y^2=\square$ ......(1), and $x^2+4xy+y^2=\square$ ......(2). Let $x=my$, then from (1) $m^2+1=\square=$(say) $(p-m)^2$, from which $m=(p^2-1)/2p$. From (2), $m^2+4m+1=\square=$(say) $(pm-1)^2$. $m=(2p+4)/(p^2-1)=(p^2-1)/2p$, or $(p^2-1)^2=4p^2+8p$. Hence $4p^2+8p=\square$. Or $p^2+2p=\square=$(say) $q^2-2pq+p^2$, and $p=q^2/[2(q+1)]$. Then $4p^2+8p=[(q^2+2q)/(q+1)]^2$, and $p^2-1=(q^2+2q)/(q+1)$, and $p^2=(q^2+3q+1)/(q+1)$;

$$p=\sqrt{[(q^2+4q^2+4q+1)]}/(q+1).$$

The only methods, which I know, of making the numerator rational, give $q=0$, and $p=1$, and $m=0$.

Taking $p^2+2p=p^2q^2$ and proceeding in a similar manner as above, we get $p=[1/(q^2-1)]\sqrt{(q^4+4q^3-2q^2-4q+1)}$, and we get $q=0$, and $p=-1$ or $q=-1$ and $p=0$.

I have tried many other methods and all give $p=1$, or $p=-1$. While all this does not *demonstrate* that the question is impossible, I shall believe that it is so, until I see a solution.

63. Proposed by A. H. HOLMES, Brunswick, Me.

Given $x^3+y^3=20^3\times105489$, to find four positive *integral* values each for x and y.